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Technical Note

Feedback control of the instability of a fluid layer flowing down a vertical cylinder

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1. Introduction

The instability of a fluid layer flowing down a vertical cylinder has attracted many researchers due to its important applications in many industrial processes. Such processes mainly include: coating surface, absorber and reactor. For coating surface process, the liquid film should be stabilized to maintain smooth and uniform film coating. On the contrary, for absorber and reactor, the liquid film should be destabilized to improve heat and mass transfer across the free surface. Based on the above reasons, the suitable flow control strategy should be developed to delay or enhance the instability of liquid layer.

Goren [1] studied the linear stability of external and internal films with respect to axisymmetric disturbances. However, the study was severely limited by assuming the zero velocity of basic flow. He found the liquid film is unstable for small wave numbers, but stable for large ones. Lin and Liu [2] obtained an evolution equation by applying the small wave number and thin film approximations. An explicit condition under which a film of a constant thickness can be attained was given in terms of relevant physical parameters. Their results were in a good agreement with the known experimental results. Krantz and Zollars [3] started from the Orr–Sommerfeld equation and showed the results for small wave number and low Reynolds number. The valid domain of their asymptotic solution was established. Solorio and Sen [4] calculated the linear isothermal stability with fully developed base flow by direct numerical computation. Their results showed that the cylindrical falling film is unstable for all Reynolds numbers, Weber numbers and radius ratios. All above research is in a frame of linear stability theory. For the non-linear case, which is out of the scope of present study, investigations were performed by Shlang and Sivashinsky [5], Trifonov [6] and Hung et al. [7].

Numerous studies showed that the instability of a flow can be greatly delayed or enhanced by flow control technology. Now this technology has been widely applied in both research and engineering problems. Thus, Gad-el-Hak [8] pointed out that flow control is perhaps more hotly pursued by scientists and engineers than any other areas of fluid mechanics. Many flow control strategies such as heating or cooling, suction or injection, flexible surface, compliant coating and large-eddy breaking have been developed recently, see the review papers of Morkovin and Reshotko [9] and Gad-el-Hak [8] for details.

An efficient active feedback control loop by periodical heating was first performed by Liepmann et al. [10]. In their boundary layer experiments in water, two heating elements are used: one for exciting the T–S wave and the other for cancelling the T–S wave. They found that the localized periodical surface heating can either reduce or enhance the overall level of flow field fluctuations. In addition, by measuring the upstream wall shear stress of the controlling surface, they were able to synthesize a signal to drive the cancellation disturbance at the controlling surface. A feedback control technique was established. Furthermore, they demonstrated that the energy cost for controlling flow can be greatly reduced by applying this technique.

Recently, Bau [11] presented a non-intrusive feedback control strategy to control the Marangoni–Bénard convection of an infinite fluid layer, where a sensor detects the deviation of the free surface temperature from its conductive value and a actuator modifies the heated wall temperature according to a linear rule of the sensor output. He showed the Marangoni–Bénard convection can be controlled powerfully. In this note, another

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Nomenclature

a	general quantity
$\hat{a}$	shape function of a
Bi	Biot number $\frac{H^* h^*}{\kappa^*}$
$\hat{c}$	complex eigenvalue
c_i	imaginary part of $\hat{c}$
c_p	specific heat at constant pressure
c_r	phase velocity
f	controller function
g	gravity acceleration constant
h^*	unperturbed fluid thickness
H	heat transfer coefficient
K	controller gain
M	number of polynomials for $\hat{\psi}$
N	number of polynomials for $\widehat{T}$
Pr	Prandlt number $\frac{\bar{\mu}_w^* c_p^*}{\kappa^*}$
Re	Reynolds number $\frac{\rho^* \bar{w}_s^* h^*}{\bar{\mu}_w^*}$
t	time
T	temperature
$\widehat{T}$	temperature shape function
w	streamwise velocity
We	Weber number $\frac{\sigma^*}{\rho^* h^* \bar{w}_s^{*2}}$
r, θ, z	Cylindrical coordinates

Greek symbols

α	wave number
β	dimensionless radius $\frac{R^*}{h^*}$
γ	the rate of radius $\frac{\beta}{\beta+1}$
η	disturbed free surface displacement
Θ	controller phase angle
κ	thermal conductivity
μ	viscosity
ρ	density
σ	surface tension
ψ	stream function
$\hat{\psi}$	shape function of stream function

Superscripts

$*$	dimensional quantity
$-$	mean value
$\sim$	disturbance quantity
s	optimal controller phase angle for stabilizing
d	optimal controller phase angle for destabilizing

Subscripts

s	free surface
w	wall

non-intrusive feedback control strategy is presented. The results show that this control strategy can greatly stabilize or destabilize a fluid layer flowing down a vertical cylinder.

2. Linear stability equations

The system whose stability will be controlled is a fully developed fluid layer flowing down a vertical cylinder as shown in Fig. 1. All fluid properties (density, thermal conductivity and specific heat) are assumed to be constant except the temperature-dependent viscosity. This is a very good approximation for some liquids like water. Cylindrical coordinates are used here. z and r are in the vertical downward and in the radial direction, respectively. The unperturbed thickness of fluid layer is h^* and the radius of cylinder is R^*. The gravitational acceleration $\vec{g}^*$ is in the z-direction.

In linear stability theory, a quantity a^* (stream function ψ^*, temperature T^* and viscosity μ^*) is decomposed into a basic (mean) value $\bar{a}^*$ and a superimposed disturbance $\tilde{a}^*$ as

$$a^* = \bar{a}^* + \tilde{a}^*. \quad (1)$$

Then we non-dimensionalize all quantity and control equations by the film thickness h^*, the basic velocity at

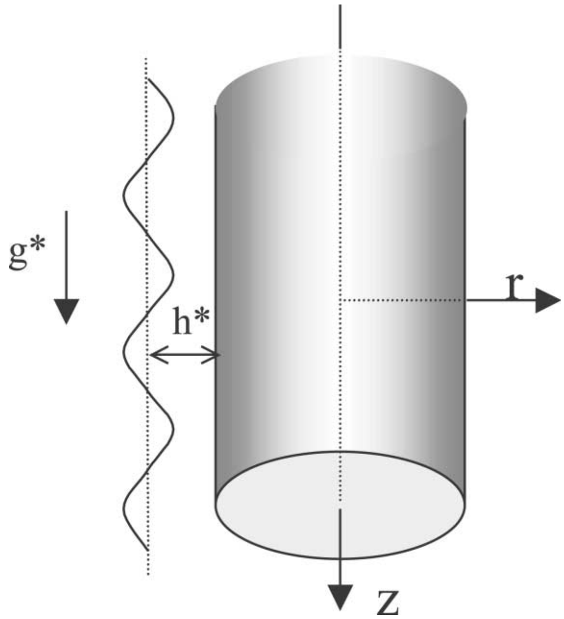

Fig. 1. Geometry of the fluid layer falling down a cylinder.

free surface $\bar{w}_s^*$, the basic temperature at the cylinder wall $\overline{T}_w^*$ and the liquid density ρ^*. The quantity with $*$ is dimensional.

In this note, we shall describe a non-intrusive feedback control strategy in order to stabilize or destabilize a fluid layer flowing down a vertical cylinder. In this method, an optical sensor detects the displacement of disturbed free surface and a actuator modifies the wall temperature of the cylinder according to a linear function of the optical sensor output. In the structure of linear stability, the controlled temperature fluctuation introduced by the actuator at the cylinder wall should not have any influences on the constant basic temperature field. The temperature-dependent viscosity is expressed as $\mu = \bar{\mu}(\overline{T}) + \tilde{\mu}(\overline{T}, \widehat{T})$ and the basic dimensionless temperature and viscosity are

$$\overline{T} = \frac{\overline{T}^*}{\overline{T}_w^*} = 1, \quad \bar{\mu}(\overline{T}) = \frac{\bar{\mu}^*}{\bar{\mu}_w^*} = 1, \tag{2}$$

where the $\overline{T}_w^*$ and $\bar{\mu}_w^*$ are the basic temperature and viscosity at the cylinder wall. The basic dimensionless flow velocity is

$$\bar{w}(r) = \frac{\gamma^2 + 2\ln\frac{r}{\beta} - \frac{r^2}{(\beta+1)^2}}{\gamma^2 - 2\ln\gamma - 1}, \tag{3}$$

where $\beta = R^*/h^*$ is the dimensionless radius and $\gamma = \beta/(\beta+1)$ is the ratio of the cylinder radius divided by the fluid layer curvature radius, see Davalos-Orozco and You [12].

For the linear stability studied here, we can take a linear approximation for viscosity fluctuation as

$$\tilde{\mu}(\overline{T}, \widetilde{T}) = \mu_1(\overline{T})\widetilde{T}. \tag{4}$$

The disturbance in Eq. (1) is assumed to be axisymmetic as:

$$\tilde{a} = \hat{a}(r)\exp[\mathrm{i}\alpha(z - \hat{c}t)] + c.c., \tag{5}$$

where α is the wave number and $\hat{c} = c_r + \mathrm{i}c_i$ is a complex number, whose real part c_r is the phase velocity and the imaginary part c_i multiplied by the wave number $\omega_i = \alpha c_i$ is the growth rate. When $\omega_i > 0$, the flow is unstable.

Substituting Eqs. (1)–(5) into the Navier–Stokes equations (for temperature-dependent viscosity) and thermal energy equation, the linear differential equations for $\hat{\psi}(r)$ and $\widehat{T}(r)$ are deduced with $D^2 = r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)$

$$\begin{aligned}[(\bar{w} - \hat{c})(D^2 - \alpha^2) - D^2\bar{w}]\hat{\psi} + \frac{\mathrm{i}}{\alpha Re}(D^2 - \alpha^2)^2\hat{\psi} \\ = \frac{\mathrm{i}}{\alpha Re}\left[(D^2 + \alpha^2)\left(r\frac{\partial \bar{w}}{\partial r}\right) + \left(2rD^2\bar{w} + 4\frac{\partial \bar{w}}{\partial r}\right)\frac{\partial}{\partial r} \right. \\ \left. + r\frac{\partial \bar{w}}{\partial r}D^2\right](\mu_1\widehat{T}),\end{aligned} \tag{6}$$

$$\left[(\bar{w} - \hat{c}) + \frac{\mathrm{i}}{\alpha Re\, Pr}\left(D^2 + \frac{2}{r}\frac{\partial}{\partial r} - \alpha^2\right)\right]\widehat{T} = 0. \tag{7}$$

The no-slip boundary conditions at the cylinder wall $r = \beta$ are

$$\hat{\psi} = \frac{\partial \hat{\psi}}{\partial r} = 0. \tag{8}$$

The boundary conditions of tangential stress, normal stress and heat balances at the free surface $r = \beta + 1$ with constant surface tension are

$$\alpha^2\hat{\psi} + \frac{\partial^2\hat{\psi}}{\partial r^2} - \frac{1}{r}\frac{\partial\hat{\psi}}{\partial r} - G\frac{\hat{\psi}}{\hat{c} - 1} - \mu_1\frac{\partial\bar{w}}{\partial r}r\widehat{T} = 0, \tag{9}$$

$$\begin{aligned}\alpha(\hat{c} - 1)\frac{\partial\hat{\psi}}{\partial r} + \frac{1}{\mathrm{i}Re}\left[\frac{\partial^3\hat{\psi}}{\partial r^3} - \frac{1}{r}\frac{\partial^2\hat{\psi}}{\partial r^2} - \left(\alpha^2 - \frac{1}{r^2}\right)\frac{\partial\hat{\psi}}{\partial r}\right] \\ - \frac{\alpha We\,\hat{\psi}}{\hat{c} - 1}\left(\alpha^2 - \frac{1}{r^2}\right) + \frac{2\mathrm{i}\alpha^2}{Re}\left(\frac{\partial\hat{\psi}}{\partial r} - \frac{\hat{\psi}}{r}\right) \\ - \frac{\mu_1\widehat{T}}{\mathrm{i}Re}\left(r\frac{\partial^2\bar{w}}{\partial r^2} + \frac{\partial\bar{w}}{\partial r}\right) = 0,\end{aligned} \tag{10}$$

$$\frac{\partial\widehat{T}}{\partial r} = -Bi\,\widehat{T}, \tag{11}$$

where Bi, Pr, Re and We are Biot number, Prandtl number, Reynolds number and Weber number, respectively, and $G = 4(1-\gamma)^2/(\gamma^2 - 2\ln\gamma - 1)$. The kinematic boundary condition has been used in the boundary conditions at $r = \beta + 1$.

The left boundary condition for the temperature disturbance at the cylinder wall is assumed to be freely adjustable. The actual temperature will be prescribed according to our feedback control strategy.

3. Active feedback control strategy

Our control strategy is realized in the following way. We suppose to apply an optical sensor to measure the perturbed free surface η^*. It is possible to get η^* (from the tangent direction) because we only consider axisymmetric disturbances and the η^* value does not depend on the coordinate θ. Then, from the difference between the measured disturbed free surface η^* and the known unperturbed free surface h^*, the disturbance amplitude of free surface $\hat{\eta}$ can be determined. The cylinder wall temperature is adjusted by the actuator according to a function of $\hat{\eta}$ which is written as $f(\hat{\eta})$. Now our feedback control loop is established. f is called controller function. For a simplified case, we choose a linear function as the controller function here. Other controller functions can be adopted in the same way. Then the controlled temperature fluctuation at $r = \beta$ is expressed as

$$\widehat{T} = K e^{i\Theta}\hat{\eta} = K e^{i\Theta}\frac{\hat{\psi}|_{r=\beta+1}}{(\beta+1)(1-\hat{c})}, \tag{12}$$

where K is the controller gain. Θ is the controller phase angle which is the difference between the phase angle of the input temperature fluctuation at the cylinder wall (the output phase angle of the actuator) and that of the measured free surface displacement (the output phase angle of the optical sensor). The boundary condition (12) is our feedback control loop. It is adopted in all following calculations.

4. Description of numerical method

The system of Eqs. (6) and (7) with their corresponding boundary conditions (8)–(12) is solved by the Chebychev collocation method. In this method, $\hat{\psi}$ and $\widehat{T}$ are expanded in Chebyshev polynomials as

$$\hat{\psi}(z) = \sum_{n=0}^{M} c_n T_n(z), \quad \widehat{T}(z) = \sum_{n=0}^{N} d_n T_n(z), \tag{13}$$

here $T_n(z) = \cos(n\cos^{-1} z)$ is the Chebyshev polynomial of order n.

Substituting Eq. (13) into Eqs. (6) and (7), the resulting algebraic equations are

$$\begin{aligned}&[L_1(T_0(z)), L_1(T_1(z)), \ldots, L_1(T_M(z))][c_0, c_1, \ldots, c_M]^{\mathrm{T}}\\ &\quad + [L_2(T_0(z)), L_2(T_1(z)), \ldots, L_2(T_N(z))][d_0, d_1, \ldots, d_N]^{\mathrm{T}}\\ &\quad = 0,\end{aligned} \tag{14}$$

$$[L_3(T_0(z)), L_3(T_1(z)), \ldots, L_3(T_N(z))][d_0, d_1, \ldots, d_N]^{\mathrm{T}} = 0. \tag{15}$$

Here L_1, L_2 and L_3 are the differential operators. By applying them, Eqs. (6) and (7) are written as $L_1\hat{\psi} + L_2\widehat{T} = 0$ and $L_3\widehat{T} = 0$, respectively.

M and N can be any numbers if the desired numerical accuracy of the solutions of Eqs. (14) and (15) with their boundary conditions can be reached. For a better approximation and optimal CPU time, the same number of collocation points should be used to solve Eqs. (14) and (15). Then we have the relation $M = N + 2$. Eqs. (14) and (15) are assumed to satisfy Eqs. (6) and (7) at the carefully chosen collocation points z_j ($j = 1, 2, \ldots, M + 1 - M_b$, M_b is the number of boundary conditions of Eq. (6) and $M_b = 4$ here). Eqs. (14) and (15) at all z_j provide $2(M + 1 - M_b)$ algebraic equations. Their corresponding boundary conditions (8)–(10) and (11) and (12) provide $M_b = 4$ and $N_b = 2$ algebraic equations, respectively. The total algebraic equations are $2(M + 1 - M_b) + M_b + N_b = 2M = (M + 1) + (N + 1)$. Then, the $(M + 1) + (N + 1)$ unknowns (i.e. the coefficients c_n ($n = 0, 1, \ldots, M$) and d_n ($n = 0, 1, \ldots, N$)) in Eq. (13) can be determined.

The choice of the collocation points can be arbitrary, but for a better approximation of $\widehat{T}(z)$ with fewer collocation points, it is better to choose $z_j = \cos\theta_j$ and $\theta_j = j\pi/(M - M_b + 2)$. The above arrangement of the collocation points shows that more points are put near the cylinder wall and the free surface boundary and fewer points are put around the middle of the fluid layer.

5. Results and discussion

For all results presented here, $M = 34$ is appropriate. The results of Solorio and Sen [4] are recalculated by our computer routine. Our results are in good agreement with those of Solorio and Sen [4] (see Table 1 for details).

The controller phase angle Θ is the only parameter which shows our active control strategy is to delay or to enhance the instability of fluid layer. For stabilizing the fluid layer, two quadrants range of controller phase angle can be used and there is one optimal angle which can stabilize the fluid layer maximally under keeping the other parameters unchanged. We call this optimal angle the optimal controller phase angle for stabilizing, written as Θ^{s} here. The dependence of the Θ^{s} value on the system parameters Re, Pr, We, Bi, γ, α, μ_1 and K is studied. It is found that the Θ^{s} value depends weakly on all above parameters except the Reynolds number Re. In fact, the Θ^{s} value depends weakly on Re too, but its dependence on Re is stronger than its dependence on the other parameters. For destabilizing the fluid layer, we choose $\Theta^{\mathrm{d}} = \Theta^{\mathrm{s}} - \pi$. Here Θ^{d} is the optimal controller phase angle for destabilizing the fluid layer.

Four typical cases are shown in Figs. 2–5. In all figures, the continuous line is corresponding to the natural case $K = 0$. Fig. 2 shows the growth rate of disturbance ω_i against wave number α. For the natural case ($K = 0$), the disturbances with wave numbers less than (about) 0.4 are unstable. After the feedback control $K = 10$ is performed, the growth rate of the most unstable disturbance is greatly reduced. But the most unstable disturbance is not fully stabilized at this time. For the controlled case $K = 20$, the growth rate of all wave number disturbance is negative and the fluid layer becomes stable. Figs. 3 and 5 show the similar phenomenon.

Table 1
Eigenvalue $\hat{c}$ for different α, $Re = 1$, $We = 1$ and $\gamma = 0.5$

α	Solorio and Sen [4]	Our results
0.4	1.706106 + 0.118957i	1.706106 + 0.118957i
0.1	1.970564 + 0.070926i	1.970564 + 0.070926i
0.01	1.999700 + 0.007641i	1.999696 + 0.007641i
0.005	1.999923 + 0.003823i	1.999923 + 0.003822i

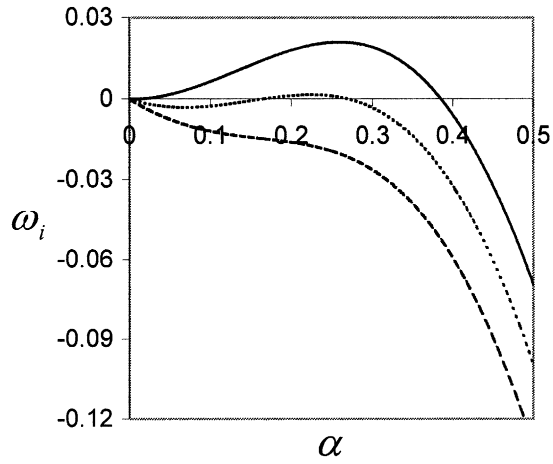

Fig. 2. The growth rate of disturbance ω_i against wave number α. The continuous line, the dashed line and the large dashed line are corresponding to $K = 0$, $K = 10$ and $K = 20$, respectively. $Re = 1$, $Pr = We = Bi = 10$, $\gamma = 0.8$, $\mu_1 = -0.02$ and $\Theta^s = 4.4$.

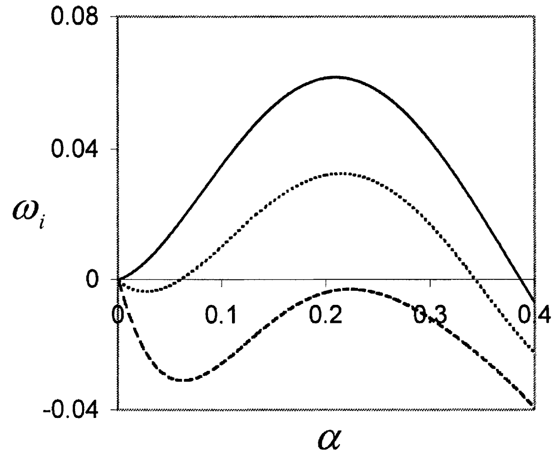

Fig. 3. The growth rate of disturbance ω_i against wave number α. The continuous line, the dashed line and the large dashed line are corresponding to $K = 0$, $K = 50$ and $K = 110$, respectively. $Re = Pr = We = Bi = 10$, $\gamma = 0.8$, $\mu_1 = -0.02$ and $\Theta^s = 3.5$.

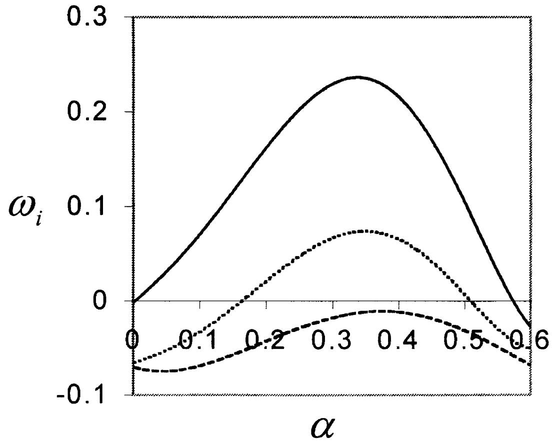

Fig. 4. The growth rate of disturbance ω_i against wave number α. The continuous line, the dashed line and the large dashed line are corresponding to $K = 0$, $K = 300$ and $K = 600$, respectively. $Re = Pr = We = Bi = 10$, $\gamma = 0.5$, $\mu_1 = -0.02$ and $\Theta^s = 3.9$.

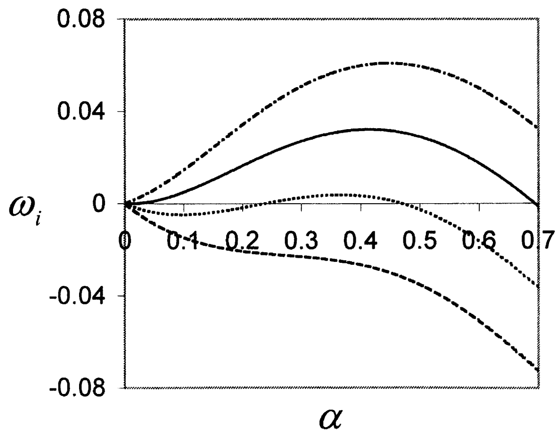

Fig. 5. The growth rate of disturbance ω_i against wave number α. $Re = We = 1$, $Pr = Bi = 10$, $\gamma = 0.99$ and $\mu_1 = -0.02$. For stabilizing case ($\Theta^s = 4.3$), the continuous line, the dashed line and the large dashed line are corresponding to $K = 0$, $K = 10$ and $K = 20$, respectively. For destabilizing case ($\Theta^d = 1.2$), the dash-dotted-curve is corresponding to $K = 10$.

Fig. 4 shows the fluid layer becomes stable until the controller gain is about 600. It is found that the growth rate of the disturbance with small wave number is greatly reduced after applying our feedback control. This phenomenon does not appear in the other cases shown here. Further study shows it only appears when the rate of radius γ is small enough.

From the Figs. 2–5, it is found that all unstable disturbance is well suppressed by applying our active feedback control strategy. It is concluded that our feedback control strategy is very powerful in stabilizing the fluid layer flowing down a cylinder if the controller phase angle and the controller gain are suitably chosen.

Besides stabilizing the fluid layer, Fig. 5 also shows the result of destabilizing the fluid layer. The control is realized by choosing $\Theta^d = \Theta^s - \pi = 1.2$. The result shows that our active feedback control strategy is also powerful to destabilize the fluid layer.

Eq. (12) shows the relation between the amplitude of the input temperature fluctuation on the cylinder wall and the amplitude of the detected free surface deformation. We recall the dimensional form of Eq. (12), that is

$$\frac{\widehat{T}^*}{\overline{T}^*_w} = K e^{i\Theta} \frac{\hat{\eta}^*}{h^*} = K e^{i\Theta} \frac{(1-\gamma)\hat{\eta}^*}{\gamma R^*}. \qquad (16)$$

If we only concern the module of the amplitude of the input temperature fluctuation, then

$$|\widehat{T}^*| = K\overline{T}^*_{\mathrm{w}} \frac{(1-\gamma)|\hat{\eta}^*|}{\gamma R^*}. \tag{17}$$

The above relation shows the module of the amplitude of the input temperature fluctuation depends on K, $\overline{T}^*_{\mathrm{w}}$, R^*, γ and $|\hat{\eta}^*|$. For $\gamma = 0.5$, $R^* = 0.1$ m, $\overline{T}^*_{\mathrm{w}} = 300$ K and $K = 600$ which is corresponding to Fig. 4, if $|\hat{\eta}^*| = 10^{-6}$ m, the module of the amplitude of the input temperature fluctuation $|\widehat{T}^*|$ is about 1.8 K.

6. Conclusions

A non-intrusive active feedback control strategy is presented in this note. The active feedback control realized by using a optical sensor to detect the displacement of the disturbed free surface and a actuator to modify the cylinder wall temperature according to a linear function of the optical sensor output. Numerical results show that all unstable disturbance is well suppressed by applying this control strategy. It is concluded that the method is very powerful in stabilizing the fluid layer flowing down a cylinder if the controller phase angle and the controller gain are suitably chosen. On the other hand, it is also shown that this control strategy can be applied to destabilize the fluid layer by performing a reverse feedback control.

References

[1] S.L. Goren, The stability of an annular thread of fluid, J. Fluid Mech. 12 (1962) 309.

[2] S.P. Lin, W.C. Liu, Instability of film coating of wires and tubes, AIChE J. 21 (1975) 775.

[3] W.B. Krantz, R.L. Zollars, The linear hydrodynamic stability of film flow down a vertical cylinder, AIChE J. 22 (1976) 930.

[4] F.J. Solorio, M. Sen, Linear stability of a cylindrical falling film, J. Fluid Mech. 183 (1987) 365.

[5] T. Shlang, G.I. Sivashinsky, Irregular flow of a liquid film down a vertical column, J. Phys. 43 (1982) 459.

[6] Yu.Ya. Trifonov, Steady-state traveling waves on the surface of a viscous liquid film falling down on vertical wires and tubes, AIChE J. 38 (1992) 821.

[7] C.-I. Hung, C.-K. Chen, J.-S. Tsai, Weakly nonlinear stability analysis of condensate film flow down a vertical cylinder, Int. J. Heat Mass Transfer 39 (1996) 2821.

[8] M. Gad-el-Hak, Modern developments in flow control, Appl. Mech. Rev. 49 (1996) 365.

[9] M.V. Morkovin, E. Reshotsko, Dialogue on progress and issues in stability and transition research, in: D. Arnal, R. Michel (Eds.), Laminar-Turbulent Transition, Springer, Berlin, 1990.

[10] H.W. Liepmann, G.L. Brown, D.M. Nosenchuck, Control of laminar-instability waves using a new technique, J. Fluid Mech. 118 (1982) 187.

[11] H.H. Bau, Control of Marangoni-Bénard convection, Int. J. Heat Mass Transfer 42 (1999) 1327.

[12] L.A. Davalos-Orozco, X.Y. You, Three-dimensional instability of a liquid layer flowing down a heated vertical cylinder, Phys. Fluids A 12 (2000) 2198.